ELASTIC MATERIALS

TNCG13 - MODELING AND ANIMATION (REPORT 1)

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Abstract

In this paper I will concentrate on a specific method for artistic design and simulation of complex material behavior. The focus lies on the theory behind the method but not so much on the mathematical part, which will be referred to instead. The second part of the paper will be a short and casual overview of a second method which is an approach to numerical coarsening of linear elastic objects to allow for interactive and realistic physical animation of complex deformable objects. When comparing the methods, I noticed that the method used for artistic design by Martin et al. [2], differed from the one used by Kharevych et al. [1]. In fact, Martin et al. disapproved of the method used by Kharevych et al.

1 Introduction

A deformation is a change in shape. Different materials deform in different ways and simulating larger and more complex systems requires even more elaborate computational methods. Despite the evolution in hardware and faster numerical solvers, there is still one huge limitation when it comes to computer animation - the simulation costs scale with structural complexity since a fine mesh is required to capture the proper dynamical behavior of a heterogeneous object. Imagine if you have to make a detailed animation of an organ with veinal structures (Fig. 1), then the sampling required would lead to daunting simulation times since the fine scales are geometrically complex. Ignoring these fine scales can affect the overall dynamics of the object drastically which could lead to a more or less rigid body of the object or even failing to capture basic coarse deformation.

Physically based animations offers control of materials properties as a way of controlling the final deformation. By controlling the deformation of an animated object, complex material behaviors can be implied. Adding physically based secondary motion to the deformable parts, such as cloth, skin or hair, will enhance the realism of animated characters. When it comes to computer animation (creative applications), the focus mainly lies on obtaining some desired deformation and material properties are just middlemen.

2 Example-Based Elastic Materials

When different materials deform, we implicitly draw a conclusion about its underlying, constitutive material. When expanding the stockpile of possible deformations of an object, the expressive palette for physics-based computer animation can be expanded.

Most of the times, when it comes to artistic endeavors, a desired deformation is envisioned and you go from there - you know what you want and you make sure you get it that way. Quantifying material coefficients that lead to the desired deform is pretty difficult to achieve, if not impossible. In fact, just choosing a mathematical model can be daunting. As with most models, a simple one offer a few coefficients but a small expressive range. A complex model, on the other hand, have an unwieldy set of parameters.

Material models describe the connection between resulting forces and geometric deformations. Most conventional material models, though accurate, confine the creative thinking of animators since they offer limited and unwieldy control. By employing examples of desired deformations, an elastic potential is constructed. This method can be translated as describing heterogeneous, anisotropic and nonlinear materials. Even though it is fairly easy to design a set of example poses, defining a corresponding material law is a dreadful task.

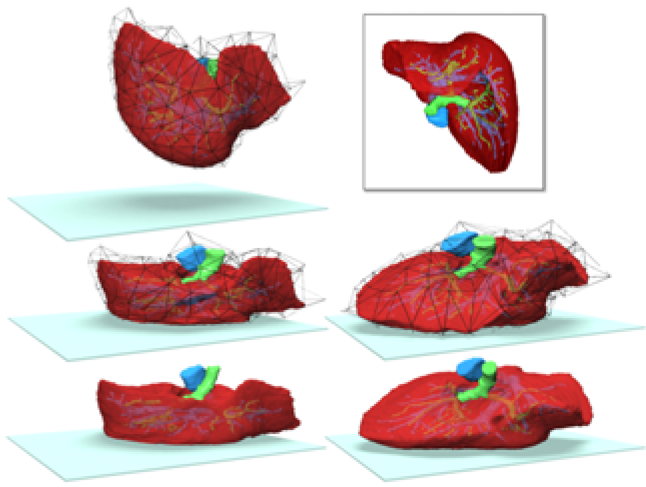

Figure 1: Numerical coarsening turns a fine mesh with heterogeneous elastic properties into a coarse mesh with anisotropic elastic properties.

Directing animations is of huge importance when it comes to setting bounds to uncontrolled physics in practical scenarios. Methods like including explicit control forces, space-time constraints and tracing approaches has been proposed for this purposes. The approach in [2] induces additional forces into the simulation. However, these forces derive from a *conservative potential*. The approach does not require a fixed plot of keyframes, it promotes a style of context-sensitive deformation control where objects are drawn to preferred shapes.

Shape Editing and Interpolation offers a lot of techniques for transferring deformations from one mesh to another and for deforming meshes. Instead of interpolating between factored deformation gradients of triangles, a nonlinear strain measure is employed, evaluated on tetrahedra. Instead of being restricted to static geometric modeling, this approach is the first to leverage example-based methods for dynamic physical simulation.

2.1 Upgrade with three novel components.

An intuitive and direct method for artistic design and simulation of complex material behavior is presented by Martin et al. [2]. Examples of characteristic desirable deformations are created from a method that accepts a set of poses. These poses are created either by hand (digitized from clay models), with a modeling tool or by taking 3D snapshots of previously run simulations.

In [2] a novel forcing term for dynamical integration is provided which causes the material to obey the "physical laws" that are implied by the providing examples.

The method presented by Martin et al. [2] offers an intuitive way to design materials with desired deformation behavior. The behavior of more complex models, such as thin shells, can also be mimicked even though the formulations are built on solid mechanics. The deformation behavior of shells can be controlled in very much the same way as for solids. This is because a surface can be embedded into a volumetric mesh, which is then deformed such that the embedded geometry assumes the desired shapes.

By incorporating three novel components, this approach can be applied to "upgrade" any existing time integration code.

2.1.1 Interpolation

A space of characteristic shapes is constructed by means of interpolation, as a result of not wanting to limit the approach to individual poses. By using a nonlinear strain measure the deformation of the example poses are quantified. This leads to a *Stain Space* which provides a rotation-invariant setting for shape interpolation. A subspace of preferable deformations is defined by the interpolated examples.

2.1.2 Projection

After the space of preferable deformations has been defined and by solving a minimization problem, configurations can be projected onto it. Its closest point on the example subspace can be computed when given an arbitrarily deformed pose.

2.1.3 Simulation

An elastic potential can be defined when combining interpolation and projection. This then attracts an object to its space of preferable deformations. A resulting point, which is the closest point to the current configuration, is extracted from each step of the animation. This point is then used as an intermediate rest configuration when computing the forces that pull the system toward the example space.

2.2 Theory

The essential idea is to define an additional elastic potential that "magnetize" a solid to its subspace of characteristic deformations, this is referred to as *example manifold*, $\mathcal{E}$.

2.2.1 Strain as a basis for the space of all deformations

Since a deformation is not affected by global rotation and translations (Fig. 2), there is no need to take these into consideration. The same goes for local rotations and translations and in the construction of nonlinear deformation measures.

2.2.2 Example Manifold

The following approach results in smooth interpolation of all elements, this is because the approach linearly interpolates the stretch and shear of each element.

$$E(w) = (1 - w)E_1 + wE_2 \tag{1}$$

E_1 and E_2 are descriptors where $E_1 = E(x_1)$ and $E_2 = E_{x_2}$ in R^{6m}. x_1 and x_2 are example poses, w denotes the interpolation weight.

Interpolated descriptors are generally not realizable, which is why a second step has to be applied to Eq. 1 to find the closet realizable strain $E(x_w) \in \mathcal{F}$ and corresponding configuration X_w by solving the least squares minimization.

$$minW_I(x_w, w) = min\frac{1}{2}|E(x_w) - E(w)|_F^2 \tag{2}$$

By using the definitions in Eq. 2 the *example manifold* $\mathcal{E} \subset \mathcal{F}$ of realizable strains.

2.2.3 Example Projection

The goal is to formulate a force that attracts x towards its projection x_w on $\mathcal{E}$. The geodesic distance on $\mathcal{F}$ between two shapes is approximated by using the elastic potential $W(X, x)$.

$$W(X, x) = \mu|E(X, x)|_F^2 + \frac{\lambda}{2}\left(\frac{V(x)}{V(X)} - 1\right)^2 \tag{3}$$

λ and μ are material coefficients and $V(\cdot)$ measures the volume of a given configuration as the sum of elemental volumes. The constrained optimization is nonlinear both in the objective function and the constraints. When applying the penalty method for constraint enforcement, we minimize

$$W_p(x_w, w, x) = W(x_w, x) + \gamma|\nabla_{x_w} W_I(x_w, w)|^2 \tag{4}$$

with respect to x_w and w, for a adequately large fixed penalty stiffness γ. The weights are constrained such that $w_i \geq 0$ and $\sum_i w_i = 1$.

2.2.4 Example-based Simulation

The system presented by Martin et al. [2] integrates willingly with the existing solid simulators. The system, however, is particularly convenient to build on top of a finite element solver, which allows reuse of code for deformation measure an elastic potentials.

To solve for static equilibrium the sum of all external forces are equal to the internal forces induced by the new, augmented material with potential. The static solution is found with the correct projection for the example potential. In order to reduce the number of unknown variables, Eq. 3 is used for both of the elastic potentials. The material constants of the example potential are set to scalar multiples of those of the conventional potential. This leaves only one unknown variable to be set.

When it comes to the dynamic simulation, it is preferred to use the implicit Euler method. The time stepping scheme is formulated as an optimization problem. It starts with the canonical equations of motion, and when applying the implicit Euler integration scheme, a nonlinear system of equations is obtained. This system can be solved by minimizing the objective function (Eq. 5) and when optimizing the solved system, the coupled problems of projection and time stepping will be solved as well.

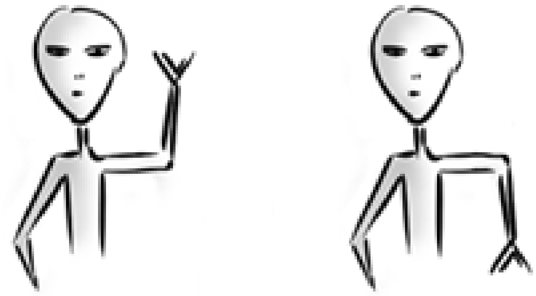

Figure 2: If parts of a solid object (the arms of a character) transform rigidly, they have (locally) not changed in shape, i.e. the shape of the arm itself has not changed even though it has been rotated or translated.

$$H(x_n, x_w, w) = \frac{h^2}{2}\left(\frac{x_n - x_0}{h^2} - \frac{v_0}{h}\right)^T M \left(\frac{x_n - x_0}{h2} - \frac{v_0}{h}\right) + W_c(x_n) + W_p(x_w, w, x_n) \quad (5)$$

2.3 Example Design and Implementation

2.3.1 Example Design

The method used in [2] relies heavily on example poses to model the characteristic deformation behavior of elastic solids. The method works with deformed element meshes which have the same topology as the undeformed mesh.

2.3.2 Embedding Triangle Meshes

To increase the level of details to impressive amounts, the use of embedding (also mentioned by Kharevych et al. [1]) is a very efficient way. It is a distinguished way for transforming a highly detailed geometry in accordance to the deformations of a much simpler approximation. The physical details, however, cannot be increased.

In order to get a desired deformable shape, the use of a static solver comes at hand. It is quite easy to deform an embedding mesh so that its enveloped surface assumes the desired shape.

By using this method it is possible to design examples that account for realistic deformations of subelement geometry, but also design examples that forge the deformation behavior of more complex mechanics.

2.3.3 Local and Global Examples

Global Examples - the deformation of one part of the object affects the entire object (Fig. 3b). This can be understood as a "*what you see is what you get*" approach to material design.

Local Examples - the deformation behavior only affects locally and independently of other regions. Local Examples can be combined to generate even more complex global behavior. An example of when local deformations occur at the same time and independently of each other is illustrated in Fig. 3a). (The shoe animation also showcases the application of embedding - the fine geometry of the shoe deforms in accordance to the coarse embedding mesh but does so in a very plausible way.)

3 Inhomogeneous Elastic Materials

During the past decade there has been a lot of papers written about how to simplify a model while still capturing (numerically or visually) its coarse physical behavior. A practical solution to a numerical coarsening problem is presented by Kharevych et al. [1]. That paper proposed a formulation of elasticity on coarse resolutions. It also proposed an approach for simulating elastic objects made of nonhomogeneous, non-isotropic materials. The paper also introduce a methodology to approximate a deformable object made of arbitrary fine structures of various linear elastic materials with a dynamically similar coarse model.

Simulations of fine, heterogeneous structures on very coarse grids are allowed because of the numerical coarsening of the material properties. Inhomogeneous and /or anisotropic materials can be realistically simulated in realtime with a

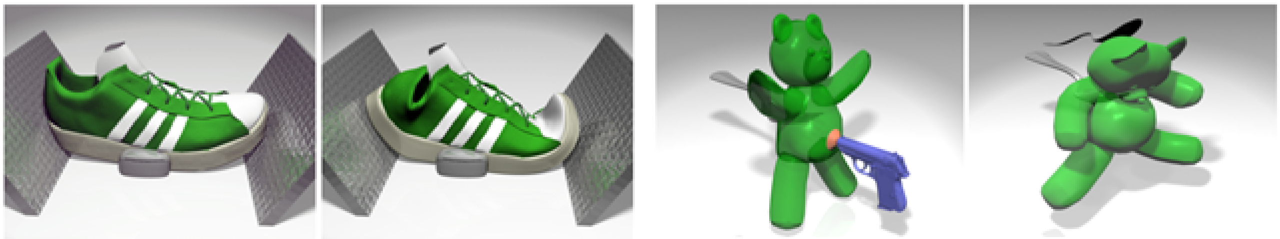

Figure 3: a) Compressed sneaker simulated as a coarse solid. Without examples to the left and augmented with two local examples to the right. b) A gummy bear is equipped with expressive examples to create an impression of personality.

numerically-coarsened model made of a few mesh elements.

Kharevych et al. [1] presents an approach to numerical coarsening of linear elastic objects which allows for interactive, realistic animation of structurally complex objects.

> "From a pair of meshes representing respectively a fine and a coarse geometric description of the elastic body, we devise a numerical procedure to turn the heterogeneous elastic properties of the fine mesh into possibly anisotropic elastic properties on the coarse mesh that effectively capture (in the H^1 sense) the same physical behavior." Kharevych et al. [1]

The coarsening procedure is achieved by computing a set of global harmonic displacements on a fine tetrahedral mesh to analyze the heterogeneous fine-scale properties, then by deducing the effective coarse-scale property for each coarse mesh element. The resulting elasticity tensors at the coarse level are often anisotropic as they reflect the object's fine, inhomogeneous composition even if the object is made out of different isotropic materials.

The coarse model is not limited to a linear space of deformations and it is simulated with a traditional finite-element solver on a coarser grid. The accuracy of this approach decays gracefully with the maximum edge length of the coarse mesh.

4 Summary

After studied **Exampled-Based Elastic Materials** by Martin et al. [2] and comparing the technique with the one used in **Elastic Materials** by Kharevych et al. [1], I noticed that most of the mathematical background is the same. There are some differences however...

Martin et al. [2] focus more on the artistic design, where characteristic desirable deformations are created from a method that accepts a set of poses. The method lets the animator design and simulate complex elastic materials as long as the animator provides a set of example poses that corresponding to characteristic, desirable or extreme deformations. To achieve the same result with a conventional simulator a tiresome tuning of an inhomogeneous, anisotropic and probably non-linear material (which is used by Kharevych et al. [1]) would be required.

Martin et al. [2] also used Global and Local Examples to make the deformation of an object affect the entire or just local parts of the object, where combining local parts will generate a complex Global Example. The method can also be used to design deformations which are difficult to generate with conventional elastic materials or when deformations clearly exceed the realm of conventional elastic materials.

The method in [1] by Kharevych et al. focus primarily at volumetric solids, but because of their embedding technique it is possible to use it to mimic the behavior of more complex mechanical models such as thin shells. The method, however, is limited to linear elasticity. The use of corotational methods injects geometric nonlinearity to coarse simulations, thus limiting the visual drawbacks of linear elasticity.

Kharevych et al. [1] tested that a homogeneous object is coarsened into the same material. They used a layered object made out of two materials and witnessed a "accordion" effect when the object was deformed perpendicular to its layers. Even though the example was simple yet anisotropic, it was enough to prove that other forms of coarsening are not enough to capture the proper dynamics on the coarse mesh.

References

[1] Lily Kharevych, Patrick Mullen, Houman Owhadi, and Mathieu Desbrun. Numerical coarsening of inhomogeneous elastic materials. *ACM Trans. Graph.*, 28:51:1–51:8, July 2009.

[2] Sebastian Martin, Bernhard Thomaszewski, Eitan Grinspun, and Markus Gross. Example-based elastic materials. *ACM Trans. Graph.*, 30:72:1–72:8, August 2011.